OTTO VON GUERICKE UNIVERSITÄT MAGDEBURG

WW FAKULTÄT FÜR WIRTSCHAFTSWISSENSCHAFT

Pricing in Global and Local Competition

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Monopoly

Assumptions

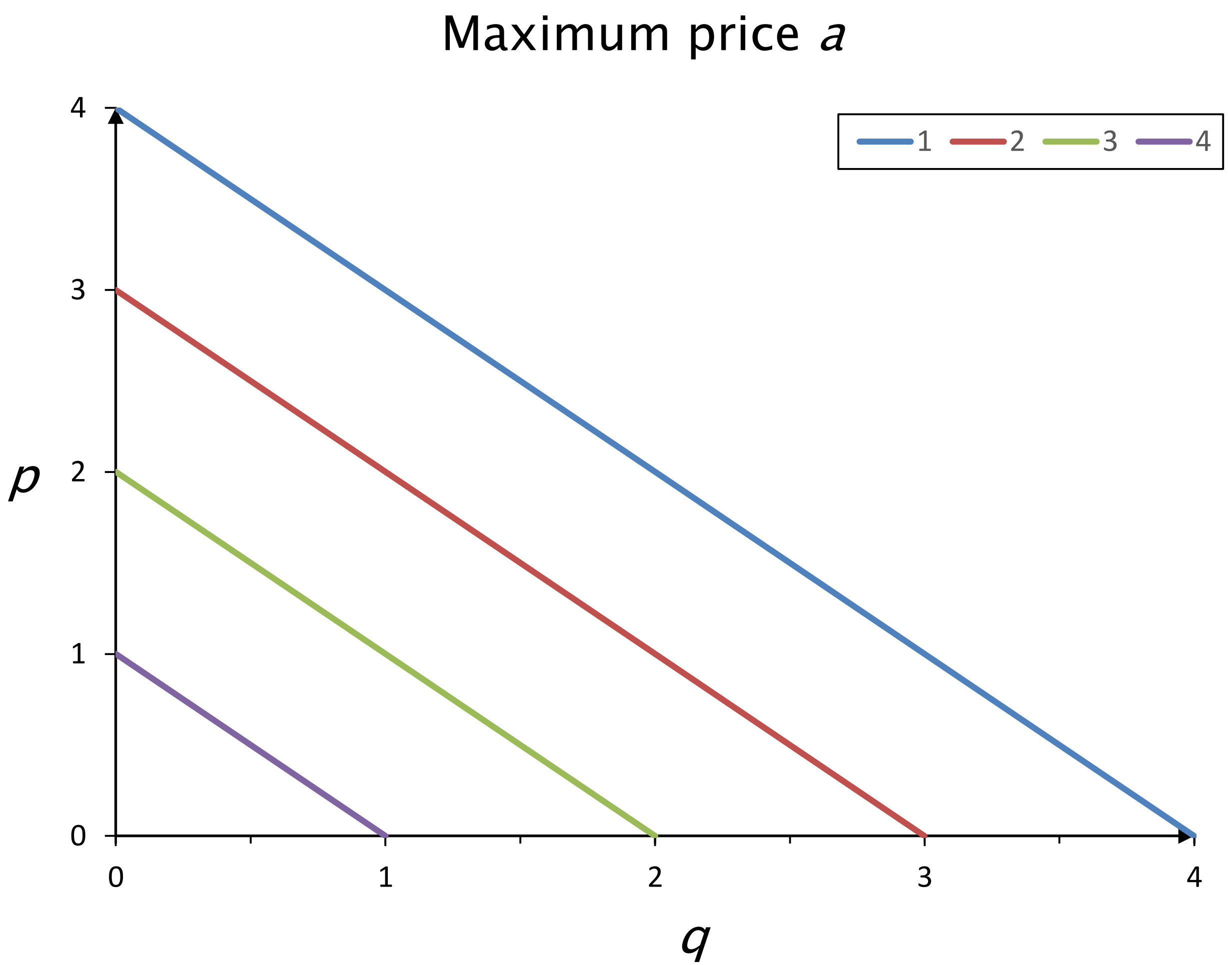
- One producer (monopolist)
- Price p, Quantity q
- Revenue function: Revenue = Price * Quantity, ($\equiv R(q,p)$)
- Cost function: $K(q,p)$
- Profit function: $\pi(q,p) = R(q,p) - K(q,p)$

Assumptions

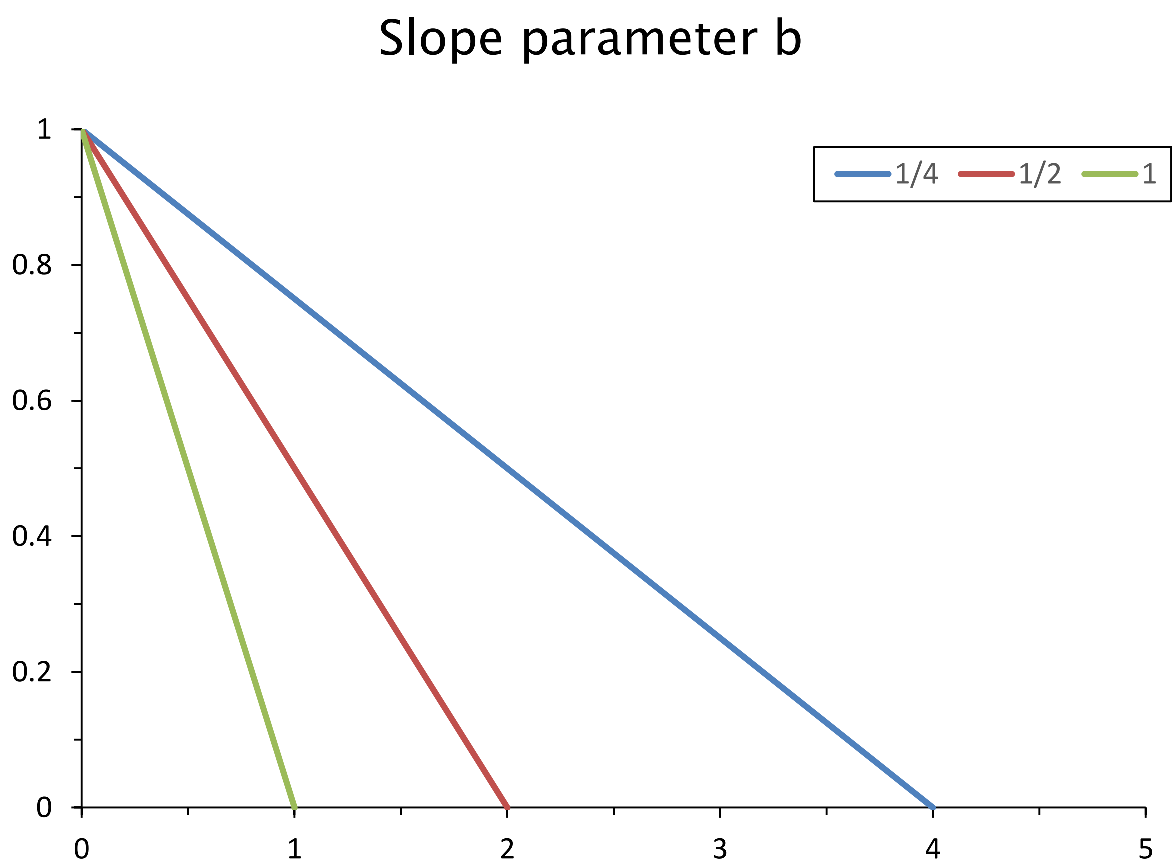
- One market
- Relationship between price and quantity
  - Demand function: $q(p) = \alpha - \beta p$
  - Inverse demand function: $p(q) = \alpha/\beta - q/\beta \equiv a - bq$
  - Parameters:

$$\alpha = a/b, \quad \beta = 1/b, \quad a = \alpha/\beta, \quad b = 1/\beta$$

Inverse demand function



Inverse demand function



Assumptions

- Goal: Maximize profits
  - Price competition
  - Quantity competition

- Assumtion: Linear cost function
  - $K(q) = kq \longrightarrow K(p) = k(\alpha - \beta p)$

Price competition

- Profit function:

$$\max_p \pi(p) = \underbrace{p(\alpha - \beta p)}_{\text{Revenue}} - \underbrace{k(\alpha - \beta p)}_{\text{Cost}}$$

- Necessary condition:

$$0 = \alpha - 2\beta p + \beta k$$

Price competition

- Results:
  - Price $p = \frac{\alpha+\beta k}{2\beta}$
  - Quantity $q = \frac{\alpha-\beta k}{2}$
  - Profit $\pi = \frac{(\alpha-\beta k)^2}{4\beta}$

Quantity competition

Assumption: $K(q) = kq$

- Profit function:

$$\max_q \pi(p) = \underbrace{q(a - bq)}_{\text{Revenue}} - \underbrace{kq}_{\text{Cost}}$$

- Necessary condition:

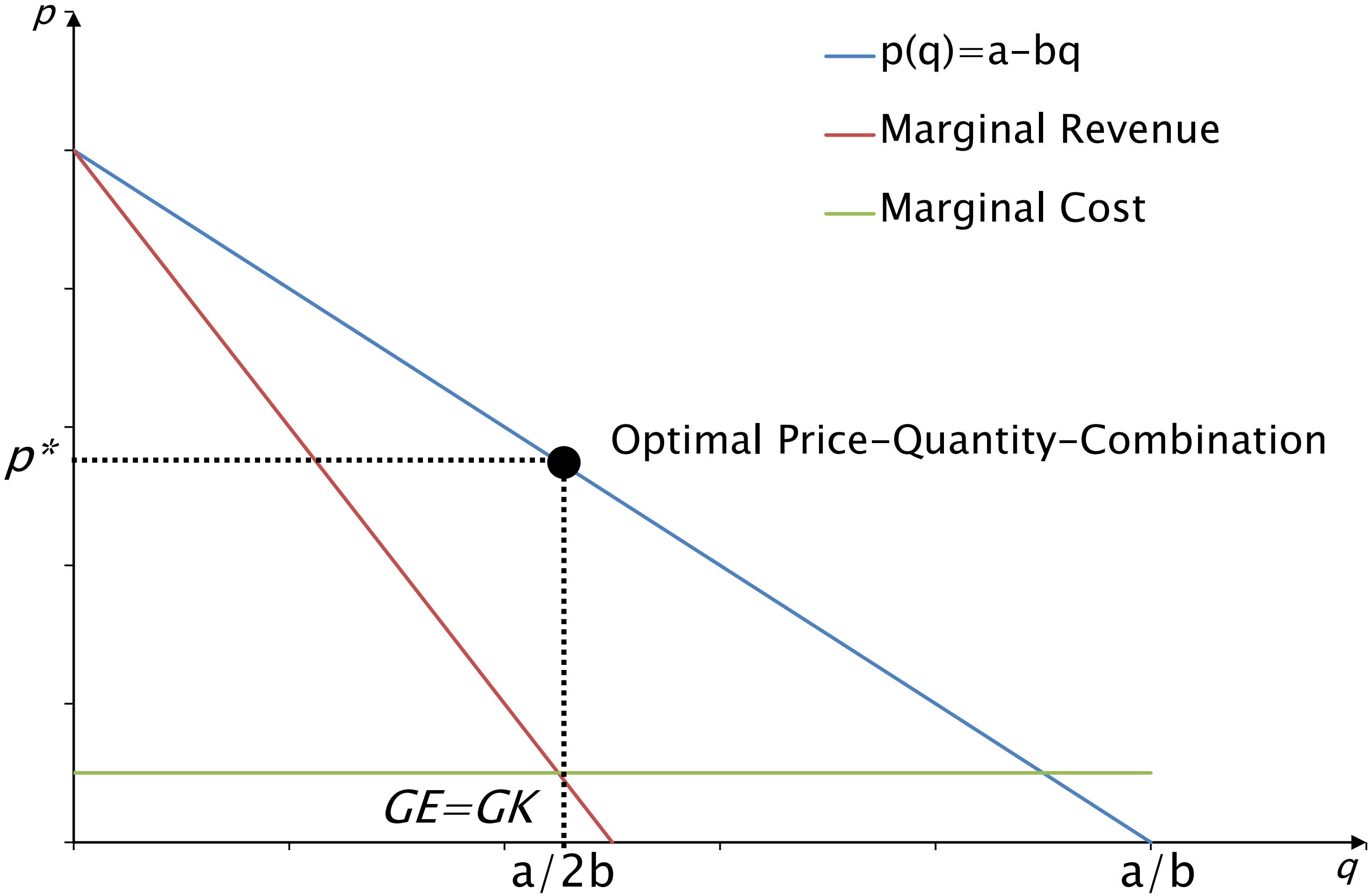
$$0 = a - 2bq + k$$

$$\underbrace{a - 2bq}_{\text{Marginal revenue}} = \underbrace{k}_{\text{Marginal cost}}$$

Quantity competition

- Results:
  - Quantity $\quad q = \frac{a-k}{2b}$
  - Price: $\quad p = \frac{a+k}{2}$
  - Profit $\quad \pi = \frac{(a-k)^2}{4b}$

Graphic analysis



Duopol

Assumptions

- Two firms ($i = 1,2$)

- Differences between products
  - Homogeneous goods
  - Heterogeneous goods

- Differences in the decision situation
  - Simultaneous decision
  - Sequential decision

- Differences in the dependent variable
  - Price competition
  - Quantity competition

Simultaneous decision

Homogeneous goods

Assumptions

- Products are not distinguishable
- Consumers always buy at the lowest price
- Firms do nothave any capacity constraint
- No collusion
- Demand function: $Q(p) = \alpha - \beta * p$
- Inverse demand function: $p(q) = a - bQ$ with $Q = q_1 + q_2$
- Cost function: $K_i = k_i q_i$
- Profit function: $\pi_i = p_i q_i - K_i$

Homogeneous goods

Quantity competition (*Cournot*)

Quantity competition

- Maximize profit

$$\max_{q_1} \pi_1(q_1) = \big(a - b(q_1 + q_2)\big)q_1 - k_1 q_1$$

- Necessary condition

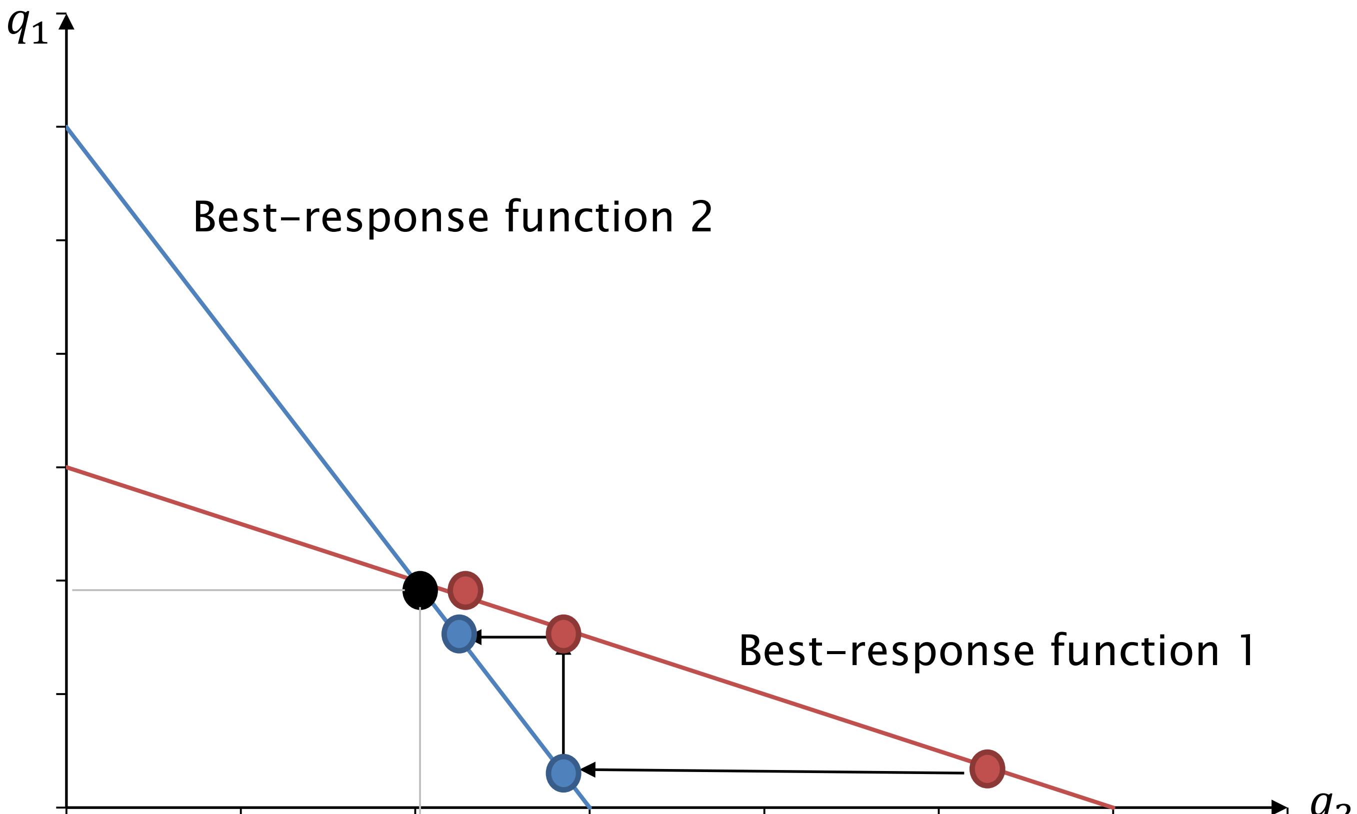
$$0 = a - 2bq_1 - bq_2 - k_1$$

- Due to symmetry, we obtain the following best-response functions

$$q_1(q_2) = \frac{a-k_1}{2b} - \frac{q_2}{2}$$

$$q_2(q_1) = \frac{a-k_2}{2b} - \frac{q_1}{2}$$

Intercept of best-response functions



Quantity competition

- Equilibrium at the intercept of the best-response functions

$$q_1(q_2) = \frac{a-k_1}{2b} - \frac{1}{2}\left(\frac{a-k_2}{2b} - \frac{q_1}{2}\right)$$

- Due to symmetry, we obtain the following quantities at the equilibrium

$$q_1 = \frac{a-2k_1+k_2}{3b}\ ;\ q_2 = \frac{a-2k_2+k_1}{3b}$$

- Total quantity

$$Q = \frac{2a-k_1-k_2}{3b}$$

Quantity competition

- Equilibrium price

$$p = \frac{a + k_1 + k_2}{3}$$

- Equilibrium profit

$$\pi_1 = \frac{(a-2k_1+k_2)^2}{9b}$$

$$\pi_2 = \frac{(a+k_1-2k_2)^2}{9b}$$

Quantity competition

Results of *Cournot*-competition

- Market price and total quantity

$$p^C = \frac{a+k_1+k_2}{3}, \; Q^C = \frac{2a-k_1-k_2}{3b}$$

- Firms

$$q_i^C = \frac{a-2k_i+k_j}{3b}; \; \pi_i^C = \frac{(a+k_1-2k_2)^2}{9b}$$

Strategic substitutes

- In the *Cournot* model, the dependent variables (Quantities) are labeled *strategic substitutes*.
- In this case, the best response to a production increase of the competing firm is a decrease in own production.

See Pepall et al. (2005, 241-243)

Homogeneous goods

Price competition (Bertrand)

Price competition

Assumptions

- Consumers buy at the lowest price
- The unit cost is k_i
- The demand depends on the price:

$$q_i = \begin{cases} 0, & \text{if } p_i > \frac{\alpha}{\beta} \text{ or } p_i > p_j; \\ \frac{1}{2}(\alpha - \beta p_i), & \text{if } p_i = p_j < \frac{\alpha}{\beta}; \\ \alpha - \beta p_i, & \text{if } p_i < p_j \text{ and } p_i < \frac{\alpha}{\beta} \end{cases}$$

Price competition

Equilibrium

- Undercutting of prices until unit costs are reached
- $k_i = k_j = k$

  Bertrand paradox: $p_i = p_j = k$. Price competition leads to the same result as perfect competition

- $k_i < k_j$

  i undercuts *j* marginally and serves the entire demand:

  $\mathrm{p} = k_j - \varepsilon(\varepsilon \to 0)$

- $k_i > k_j$

  Prices decrease until *i* cannot lower prices anymore because of the unit cost. Hence, *i* is not producing.

Price competition

Results of Bertrand-competition

- Market price and total quantity

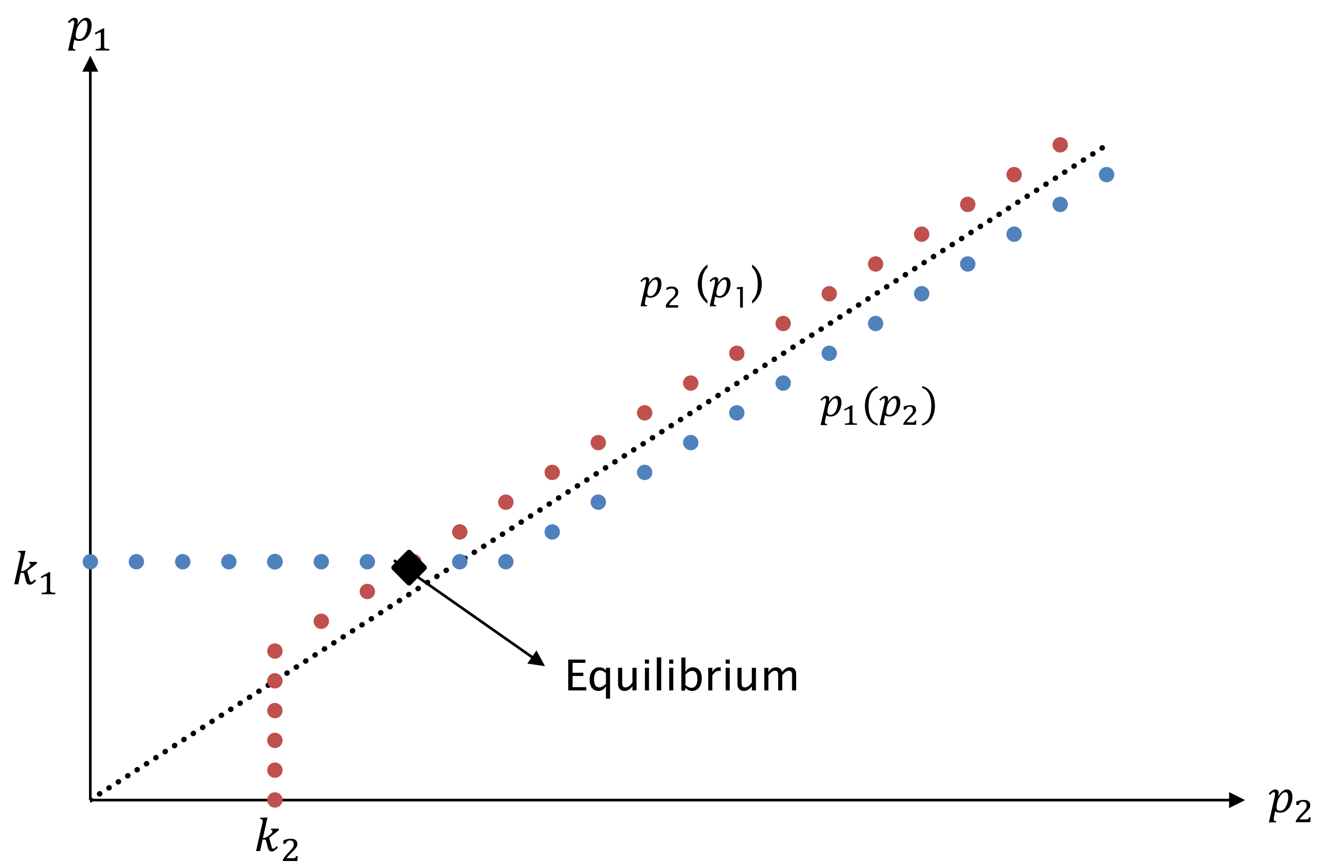
$$p^B = \max\{k_i, k_j\}; \quad Q^B = \alpha - \beta max\{k_i, k_j\}$$

- Firms

$$q_i^B = \begin{cases} 0, & \text{if } k_i > \frac{\alpha}{\beta} \text{ or } k_i > k_j; \\ \frac{1}{2}(\alpha - \beta k_i), & \text{if } k_i = k_j < \frac{\alpha}{\beta}; \\ \alpha - \beta p_i, & \text{if } k_i < k_j \text{ and } k_i < \frac{\alpha}{\beta} \end{cases}$$

$$\pi_i^B = \begin{cases} 0, & \text{if } k_i > \frac{\alpha}{\beta} \text{ or } k_i > k_j; \\ (\alpha - \beta k_j)(k_j - k_i), & \text{if } k_i < k_j \text{ and } k_i < \frac{\alpha}{\beta} \end{cases}$$

Intercept of the best-response functions



Strategic complements

- In the Bertrand model, the dependent variables (Price) are labeled *strategic complements*.
- In this case, the best response to a price increase of the competing firm is to increase the own price as well.

See Pepall et al. (2005, 241-243)

Heterogeneous goods

Assumptions

- 2 products are quasi-substitutes
- Firms do not have any capacity constraint
- No collusion
- Demand function:

$$q_1 = \alpha - \beta p_1 + \gamma p_2,\ q_2 = \alpha - \beta p_2 + \gamma p_1$$

- Inverse demand function

$$p_1 = a - bq_1 - dq_2,\ p_2 = a - bq_2 - dq_1$$

- Parameters:

$$\alpha = \frac{a(b-d)}{b^2-d^2},\ \beta = \frac{b}{b^2-d^2},\ \gamma = \frac{d}{b^2-d^2}$$

Assumptions

- Own price effect (b) vs. Cross price effect (d)
  - $b^2 > d^2$ and $b > 0$
  - Products are perfect substitutes when $b^2 = d^2$
  - Products are independent of each other when $d^2 = 0$
- Cost function: $K_i = k_i q_i$
- Profit function: $\pi_i = p_i q_i - K_i$

Heterogeneous goods

Quantity competition

Quantity competition

- Maximize Profits

$$\max_{q_1} \pi_1(q_1) = (a - bq_1 - dq_2)q_1 - k_1 q_1$$

- Necessary condition:

$$0 = a - 2bq_1 - dq_2 - k_1$$

- Due to symmetry, we obtain the following best-response functions

$$q_1(q_2) = \frac{a - dq_2 - k_1}{2b}$$

$$q_2(q_1) = \frac{a - dq_1 - k_2}{2b}$$

Quantity competition

- Equilibrium at the intercept of the best-response function

$$q_1(q_2) = \frac{a - d\left(\frac{a - dq_1 - k_2}{2b}\right)k_1}{2b}$$

- Due to symmetry, we obtain the following quantities in equilibrium

$$q_1 = \frac{(2b-d)a - 2bk_1 + dk_2}{4b^2 - d^2}$$

$$q_2 = \frac{(2b-d)a - 2bk_2 + dk_1}{4b^2 - d^2}$$

Quantity competition

- Equilibrium prices

$$p_1 = \frac{b(2b-d)a+\left(2b^2-d^2\right)k_1+bdk_2}{4b^2-d^2}$$

$$p_2 = \frac{b(2b-d)a+\left(2b^2-d^2\right)k_2+bdk_1}{4b^2-d^2}$$

- Equilibrium profits

$$\pi_1 = b\left(\frac{(2b-d)a-2bk_1+dk_2}{4b^2-d^2}\right)^2$$

$$\pi_2 = b\left(\frac{(2b-d)a-2bk_2+dk_1}{4b^2-d^2}\right)^2$$

Comparative statics

- For $k_1 = k_2 = 0$, we obtain the following results

$$q_i^C = \frac{a}{2b+d}$$

$$p_i^C = \frac{ab}{2b+d}$$

$$\pi_i^C = \frac{a^2 b}{(2b+d)^2}$$

- Variations of the parameters
  - A greater a shifts the demand curve outward and leads to greater quantities, prices, and profits
  - A greater differentiation ($b^2 - d^2$ increases) leads to greater quantities, prices, and profits (extreme differentiation: 2 Monopolies)

Heterogeneous goods
Price competition

Price competition

- Maximize Profits

$$\max_{p_1} \pi_1(p_1) = (p_1 - k_1)(\alpha - \beta p_1 + \gamma p_2)$$

- Necessary condition:

$$0 = \alpha - 2\beta p_1 - \gamma p_2 - \beta k_1$$

- Due to symmetry, we obtain the following best-response functions

$$p_1(p_2) = \frac{a+\beta k_1}{2\beta} + \frac{\gamma}{2\beta} p_2$$

$$p_2(p_1) = \frac{a+\beta k_2}{2\beta} + \frac{\gamma}{2\beta} p_1$$

Price competition

- Equilibrium at the intercept of the best-response function

$$p_1(p_2) = \frac{\alpha+\beta k_1}{2\beta} + \frac{\gamma}{2\beta}\left(\frac{\alpha+\beta k_2}{2\beta} + \frac{\gamma}{2\beta} p_1\right)$$

- Due to symmetry, we obtain the following quantities in equilibrium

$$p_1 = \frac{\alpha}{2\beta-\gamma} + \frac{\beta(2\beta k_1+\gamma k_2)}{4\beta^2-\gamma^2}$$

$$p_2 = \frac{\alpha}{2\beta-\gamma} + \frac{\beta(2\beta k_2+\gamma k_1)}{4\beta^2-\gamma^2}$$

Price competition

- Equilibrium prices

$$q_1 = \frac{\alpha\beta}{2\beta-\gamma} + \beta\,\frac{\beta\gamma k_2-(2\beta^2-\gamma^2)k_1}{4\beta^2-\gamma^2}$$

$$q_2 = \frac{\alpha\beta}{2\beta-\gamma} + \beta\,\frac{\beta\gamma k_1-(2\beta^2-\gamma^2)k_2}{4\beta^2-\gamma^2}$$

- Equilibrium profits

$$\pi_1^B = \beta\left(\frac{\alpha}{2\beta-\gamma} + \frac{\beta\gamma k_2-(2\beta^2-\gamma^2)k_1}{4\beta^2-\gamma^2}\right)^2$$

$$\pi_2^B = \beta\left(\frac{\alpha}{2\beta-\gamma} + \frac{\beta\gamma k_1-(2\beta^2-\gamma^2)k_2}{4\beta^2-\gamma^2}\right)^2$$

Comparative statics

- For $k_1 = k_2 = 0$ we obtain the following results

$$p_i^B = \frac{a(b-d)}{2b-d}$$

$$q_i^B = \frac{ab}{(2b-d)(b+d)}$$

$$\pi_i^B = \frac{a^2 b(b-d)}{(2b-d)^2(b+d)}$$

- Variations of the parameters
  - A greater a shifts the demand curve outward and leads to greater quantities, prices, and profits
  - A greater differentiation ($b^2 - d^2$ increases) leads to greater quantities, prices, and profits (extreme differentiation: 2 Monopolies)

Cournot vs. Bertrand

Prices

- Prices in Cournot competition are greater than in Bertrand competition

$$p^C - p^B = \frac{ab}{2b+d} - \frac{a(b-d)}{2b-d} = \frac{ad^2}{4b^2-d^2} > 0$$

Quantities

- Quantities in Cournot competition are lower than in Bertrand competition

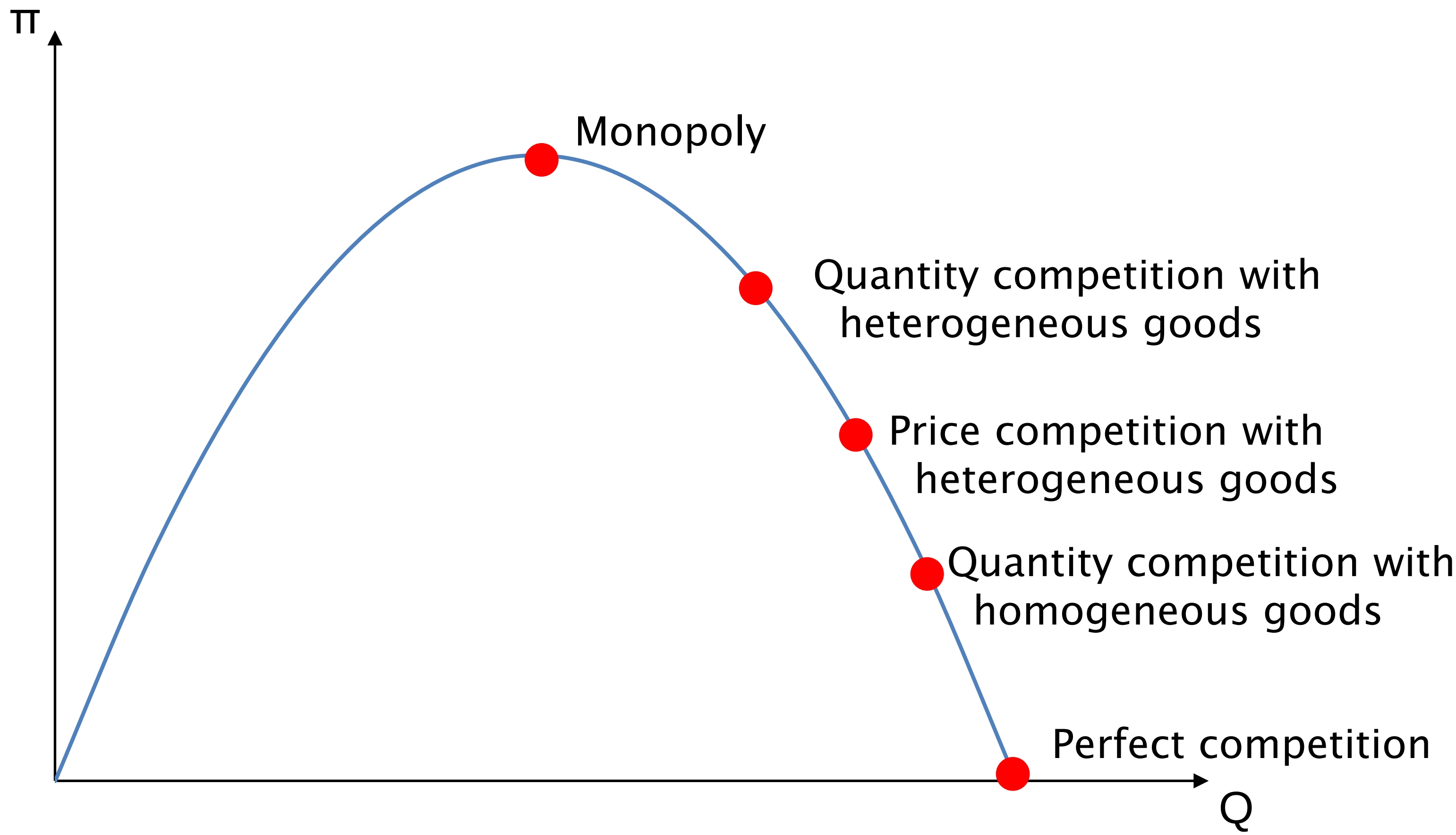
$$q^C - q^B = \frac{a}{2b+d} - \frac{ab}{(2b-d)(b+d)} = \frac{-ad^2}{(4b^2-d^2)(b+d)} < 0$$

Profits

- Profits in Cournot competition are greater than in Bertrand competition

$$\pi^C - \pi^B = \frac{a^2b}{(2b+d)^2} - \frac{a^2b(b-d)}{(2b-d)^2(b+d)} = \frac{2a^2bd^3}{(4b^2-d^2)(b+d)} < 0$$

Comparison of total profits



- The order between “Price competition with heterogeneous goods” and “Quantity competition with homogeneous goods” is not certain. There are factor combinations that lead to a greater profit for “Price competition with heterogeneous goods” compared to “Quantity competition with homogeneous goods”